Interplay between Gravity and Quantum Physics from the Point of View of General Local Covariance

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Rome, 11.07.2013

Outline of the talk

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- R. Brunetti, K. Fredenhagen, K. R., *Quantum gravity from the point of view of locally covariant quantum field theory*, [arXiv:math-ph/1306.1058]

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- Tanti Auguri Roberto!

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- Understand the small scale structure of spacetime: relation to NCG.

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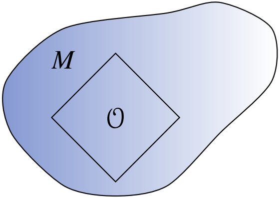

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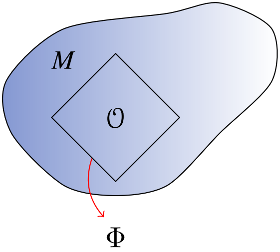

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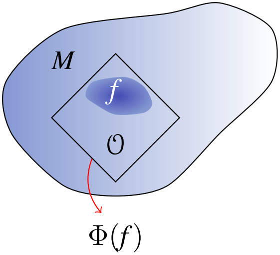

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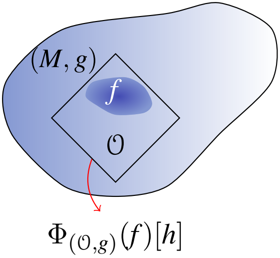

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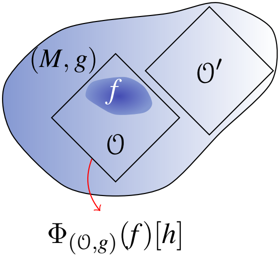
- Diffeomorphism transformation: move our experimental setup to a different region $\mathcal{O}'$.



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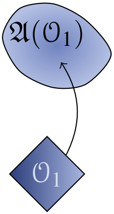

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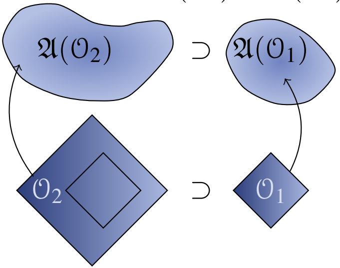

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- Mathematicaly, AQFT makes use of functional analysis techniques (operator algebras), but its various generalizations involve many other branches of mathematics.

Locally covariant quantum field theory

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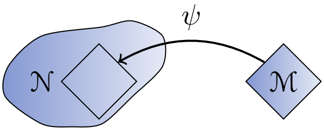
- The category **Loc**, has globally hyperbolic spacetimes $\mathcal{M} \doteq (M, g)$ as objects and its morphisms are isometric, orientations preserving, causal embeddings $\psi : \mathcal{M} \to \mathcal{N}$.



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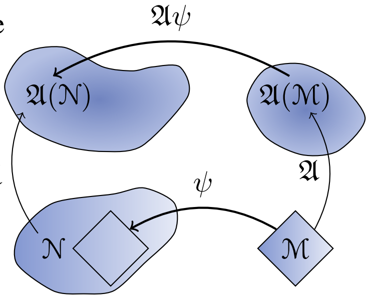
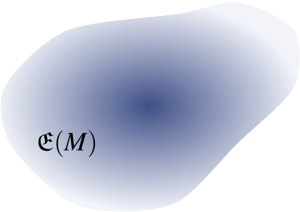
- A model in LCQFT is defined by giving a functor $\mathfrak{A}$ from the category of spacetimes to the category **Obs** of observables (for example the category of C^*-algebras).



Kinematical structure

- The configuration space (physical quantities we want to measure): space of smooth contravariant rank 2 tensors on M, $\mathfrak{E}(\mathcal{M}) = \Gamma((T^*M)^{\otimes 2})$.



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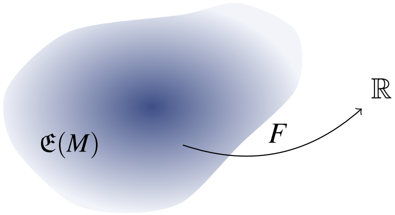

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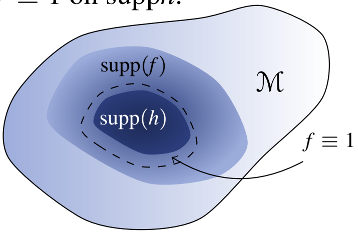
- The action $S(L)$ is an equivalence class of Lagrangians, where $L_1 \sim L_2$ if $\mathrm{supp}(L_{1,\mathcal{M}} - L_{2,\mathcal{M}})(f) \subset \mathrm{supp} df \; \forall f \in \mathfrak{D}(\mathcal{M})$.

Dynamics and symmetries

- The Euler-Lagrange derivative of S is defined by $\left\langle S'_{\mathcal{M}}(\tilde{g}), h\right\rangle = \left\langle S_{\mathcal{M}}(f)^{(1)}(\tilde{g}), h\right\rangle$, where $f \equiv 1$ on supp$h$.



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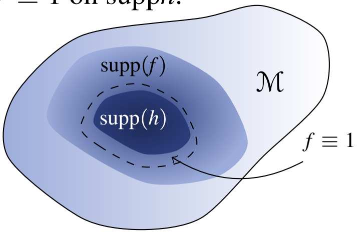

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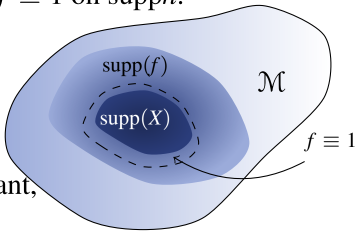

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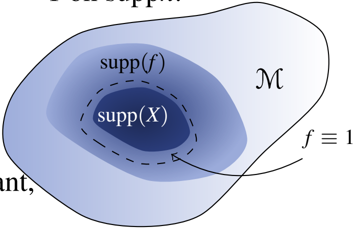
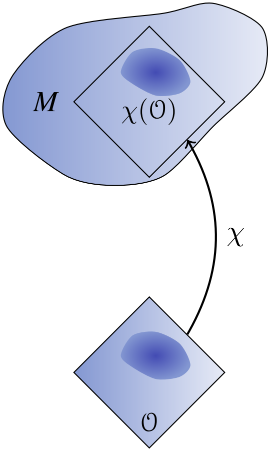


- Let $\mathfrak{E}_S(\mathcal{M})$ denote the space of solutions to field equations. We want to characterise the space of functionals on $\mathfrak{E}_S(\mathcal{M})$ which are invariant under all the local symmetries of S: invariant on-shell functionals $\mathfrak{F}_S^{\text{inv}}(\mathcal{M})$.

Fields as natural transformations

- In the framework of locally covariant field theory [Brunetti-Fredenhagen-Verch 2003] fields are natural transformation between certain functors. Let $\Phi \in \mathrm{Nat}(\mathfrak{D}, \mathfrak{F})$ (both $\mathfrak{D}$ and $\mathfrak{F}$ are treated as functors into **Vec**).



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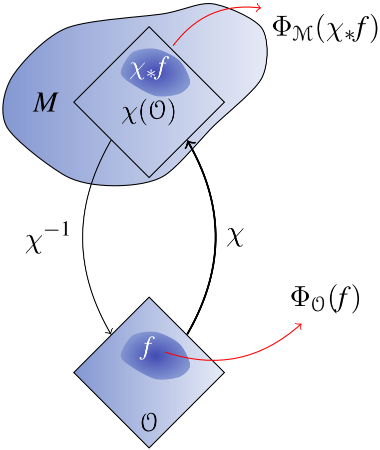

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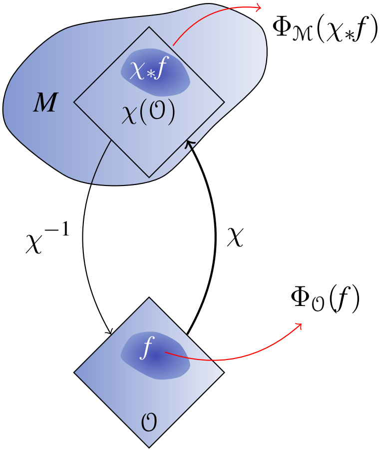
- In classical gravity we understand physical quantities not as pointwise objects but rather as something defined on all the spacetimes in a coherent way.



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- Example: $\int R[\tilde{g}] f \,\mathrm{d}\,\mathrm{vol}_{(M,\tilde{g})}$ is diffeomorphism invariant, but $\int R[\tilde{g}] f \,\mathrm{d}\,\mathrm{vol}_{(M,g)}$ is not.

Physical interpretation

- Let us fix $\mathcal{M}$. A test tensor $f \in \mathfrak{Tens}_c(\mathcal{M})$ corresponds to a concrete geometrical setting of an experiment, so for each $\mathcal{M} \in \mathrm{Obj}(\mathbf{Loc})$, we obtain a functional $\Phi(f)$, which depends covariantly on the geometrical data provided by f.

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New insight

Classical (or quantum) fields generate physical quantities, but a concrete observable quantity is obtained by evaluation on a test tensor. New concept: evaluated fields.

Evaluation of fields

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- This notion of observables corresponds to partial (relative) observables of Rovelli, Dittrich and Thiemann.

BV complex

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- The underlying algebra of the BV complex is a graded algebra denoted by $\mathcal{BV}$.

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- $\mathcal{BV}$ is geometrically interpreted as a subalgebra of the space of smooth functions on $\mathrm{Diff}_c(\mathcal{M})$ with values in multivector fields on some graded manifold $\overline{\mathfrak{E}}(\mathcal{M})$. We can equip the space of multivector fields with the Schouten bracket:

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- Gauge invariant observables are given by: $\mathcal{F}_S^{\mathrm{inv}} := H^0(s, \mathcal{BV})$.

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- Note that $H^0(s^\Psi, \alpha_\Psi(\mathcal{BV})) = H^0(s, \mathcal{BV}) = \mathcal{F}_S^{\mathrm{inv}}$.

Equations of motion and Poisson bracket

- As an output of classical field theory we have a graded manifold $\overline{\mathfrak{E}}(\mathcal{M})$ and an extended action $\tilde{S}$. Now we apply to this data the deformation quantization.

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- Using this input, we define the free Poisson bracket on $\mathcal{BV}$

$$\{F, G\}_0^g \doteq \left\langle F^{(1)}, \Delta_g G^{(1)} \right\rangle \qquad \Delta_g = \Delta_g^R - \Delta_g^A \,,$$

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- The deformation quantization of $(\mathcal{BV}_{\mu c}, \{.,.\}_0^g)$ can be performed in the standard way, by introducing a $\star$-product:

$$(F \star_H G) \doteq m \circ \exp(\hbar \Gamma_{\omega_H})(F \otimes G) \ ,$$

where $\Gamma_{\omega_H} \doteq \int dx\, dy \omega_H(x,y) \frac{\delta}{\delta\varphi(x)} \otimes \frac{\delta}{\delta\varphi(y)}$ and $\omega_H = \frac{i}{2}\Delta_g + H$ is the Hadamard 2-point function (satisfies the linearized EOM's in both arguments and the μSC).

Deformation quantization

- For a fixed $\mathcal{M}$ we have a family of algebras $\mathfrak{A}_H(\mathcal{M}) = (\mathcal{BV}_{\mu c}[[\hbar, \lambda]], \star_H)$, numbered by possible choices of H. We can define $\mathfrak{A}(\mathcal{M})$ to be an algebra consisting of families (F_H), such that $F_H = e^{\frac{\hbar}{2}\Gamma'_{H-H'}} F_{H'}$, where $\Gamma'_{H-H'} \doteq \int dx\, dy (H - H')(x, y) \frac{\delta^2}{\delta\varphi(x)\delta\varphi(y)}$ and the star product is given by

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- This leads to a deformation quantization $(\mathfrak{A}(\mathcal{M}), \star)$ of the space of evaluated fields.

Interaction

- In the next step we have to introduce the interaction, i.e. consider the algebras $\mathfrak{A}_H(\mathcal{M}) = (\mathcal{BV}_{\mu c}[[\hbar, \lambda]], \star_H)$ and define on them the renormalized time-ordered products $\cdot_{\mathcal{T}_H}$ by the Epstein-Glaser method.

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- Interacting fields are obtained from free ones by the Bogoliubov formula:

$$(R_V(\Phi))_{\mathcal{M}}(f) \doteq \frac{d}{dt}\Big|_{t=0} \mathcal{S}(V^g)^{\star-1} \star \mathcal{S}(V^g + t\Phi_{\mathcal{M}}(f)) \,.$$

Quantum observables

- In the framework of [K. Fredenhagen, K.R., CMP 2013], the S-matrix has to satisfy the so called quantum master equation (QME):

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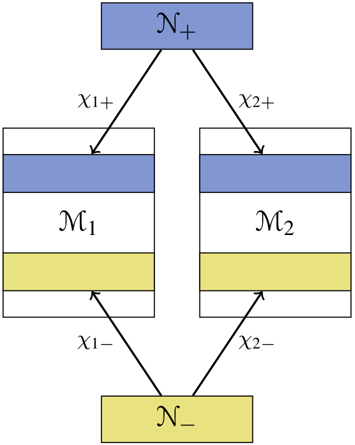
where $\triangle_{V^g}$ is the anomaly.
- If the QME holds, then gauge invariant quantum observables are recovered as the 0th cohomology of the quantum BV operator $\hat{s} \doteq R_V^{-1} \circ \{., S_0\} \circ R_V$. Equivalently,
$$\hat{s}\Phi_{\mathcal{M}}(f) = \{., S_0{}^g + V^g\} + \Phi_{\mathcal{M}}(\pounds_C f) - i\hbar\,\triangle_{V^g}\,(\Phi_{\mathcal{M}}(f))\,.$$

Relative Cauchy evolution

- Let $\mathcal{N}_+$ and $\mathcal{N}_-$ be two spacetimes that embed into two other spacetimes $\mathcal{M}_1$ and $\mathcal{M}_2$ around Cauchy surfaces, via causal embeddings given by $\chi_{k,\pm}, k = 1, 2$.



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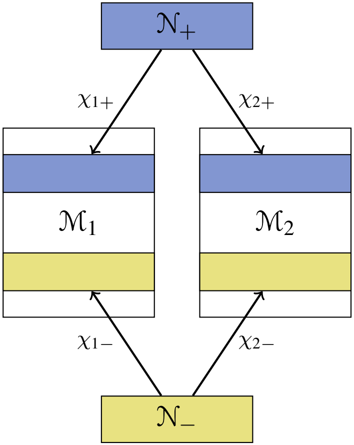

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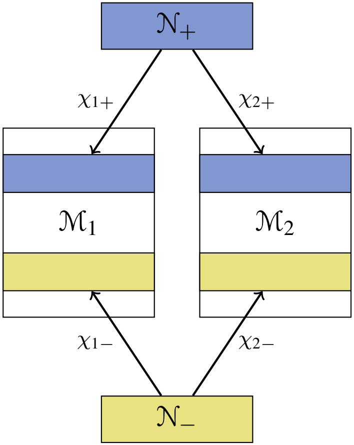
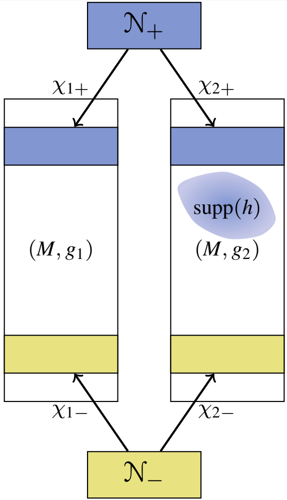


Background independence

- Let $\mathcal{M}_1 = (M, g_1)$ and $\mathcal{M}_2 = (M, g_2)$, where $(g_1)_{\mu\nu}$ and $(g_2)_{\mu\nu}$ differ by a (compactly supported) symmetric tensor $h_{\mu\nu}$ with $\mathrm{supp}(h) \cap J^+(\mathcal{N}_+) \cap J^-(\mathcal{N}_-) = \emptyset$,



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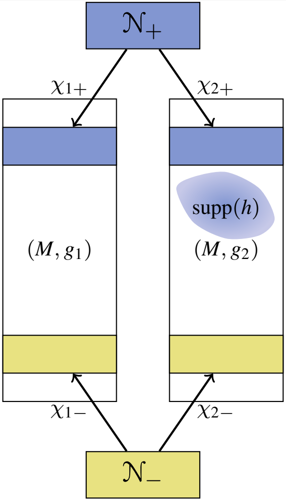

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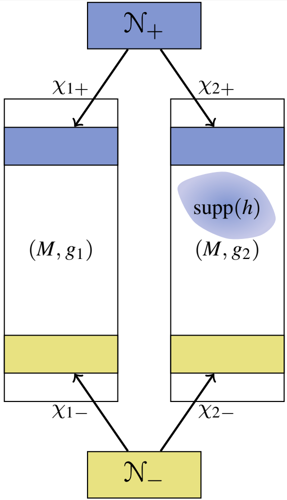
- The infinitesimal version of the background independence is condition reads: $\Theta_{\mu\nu} = 0$.



Background independence

Theorem [Brunetti, Fredenhagen, K.R. 2013]

The functional derivative $\Theta_{\mu\nu}$ of the relative Cauchy evolution can be expressed as

$$\Theta_{\mu\nu}(\Phi_{\mathcal{M}_1}(f)) \stackrel{o.s.}{=} [R_{V_1}(\Phi_{\mathcal{M}_1}(f)), R_{V_1}(T_{\mu\nu})]_\star \,,$$

where $T_{\mu\nu}$ is the stress-energy tensor of the extended action and one can define the time-ordered products in such a way that $T_{\mu\nu} = 0$ holds, so the interacting theory is background independent.

Conclusions

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- We have shown, using the relative Cauchy evolution, that our theory is background independent, i.e. independent of the split into free and interacting part.

Thank you for your attention!