Constructing Quantum Field Theories Non-perturbatively with Hamiltonian Methods

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Strongly coupled QFT

Algorithmic point of view
(we understand something when we can calculate it)

Questions:

How do we compute observable quantities in strongly coupled QFTs?

Can we improve?

Weakly vs strongly coupled QFT

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Corners of parameter space where perturbation theory is reasonable.

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E.g.: $d = 2$ $\qquad (\partial\phi)^2 + m^2\phi^2 + g :\phi^4: \qquad g/m^2 \ll 1$

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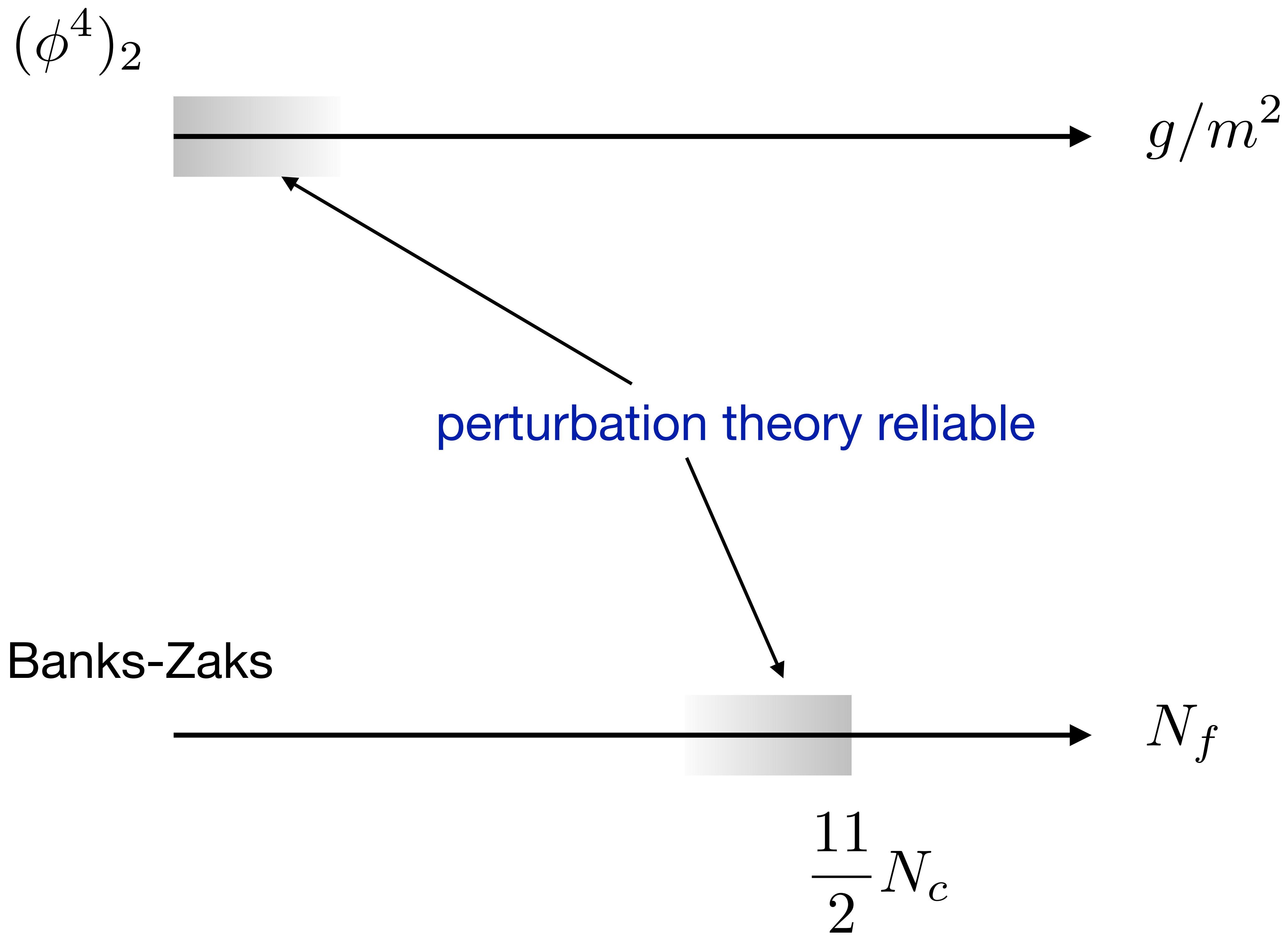
2. Scale invariant theories close to the gaussian FP

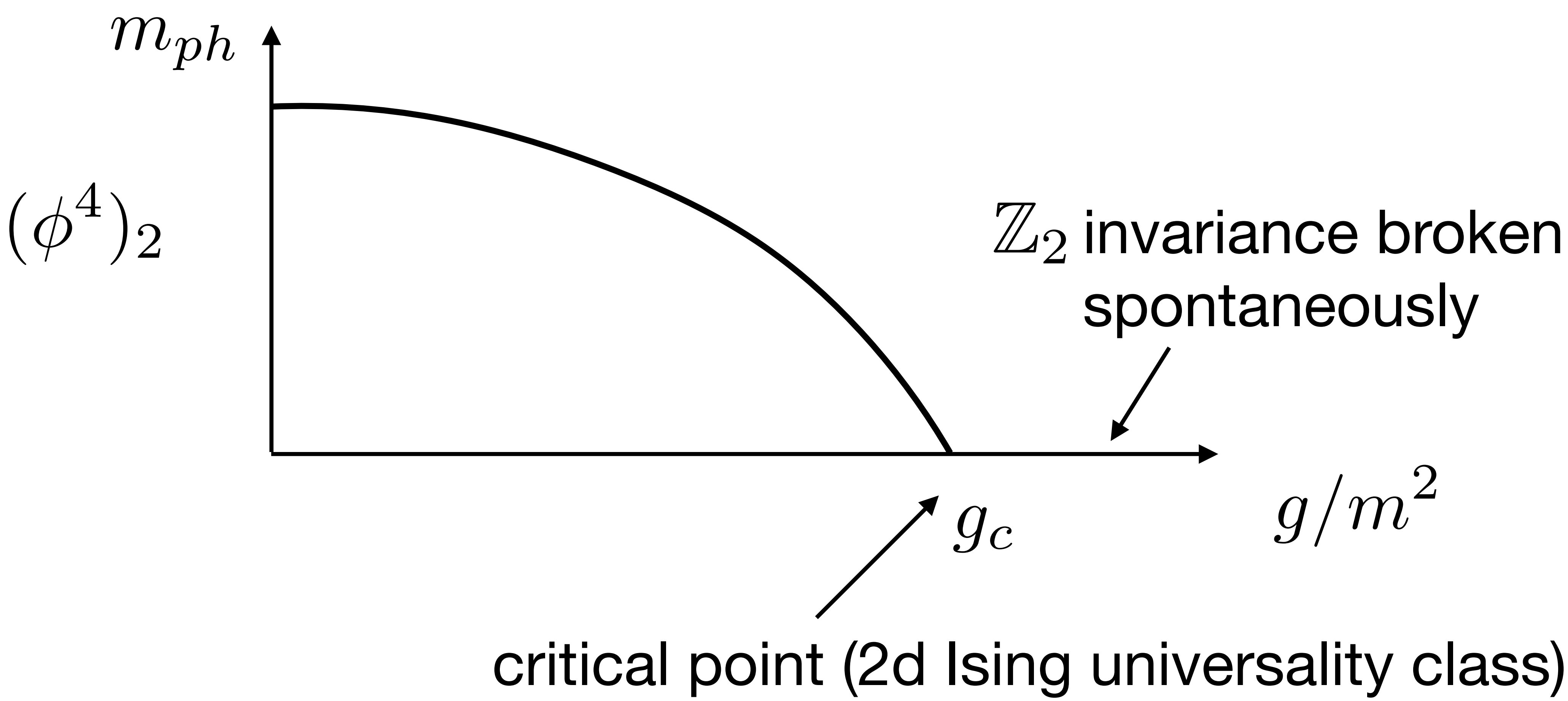
E.g.: $SU(N_c)$ gauge theory $\quad N_c \gg 1$

N_f massless Dirac fermions in the fundamental

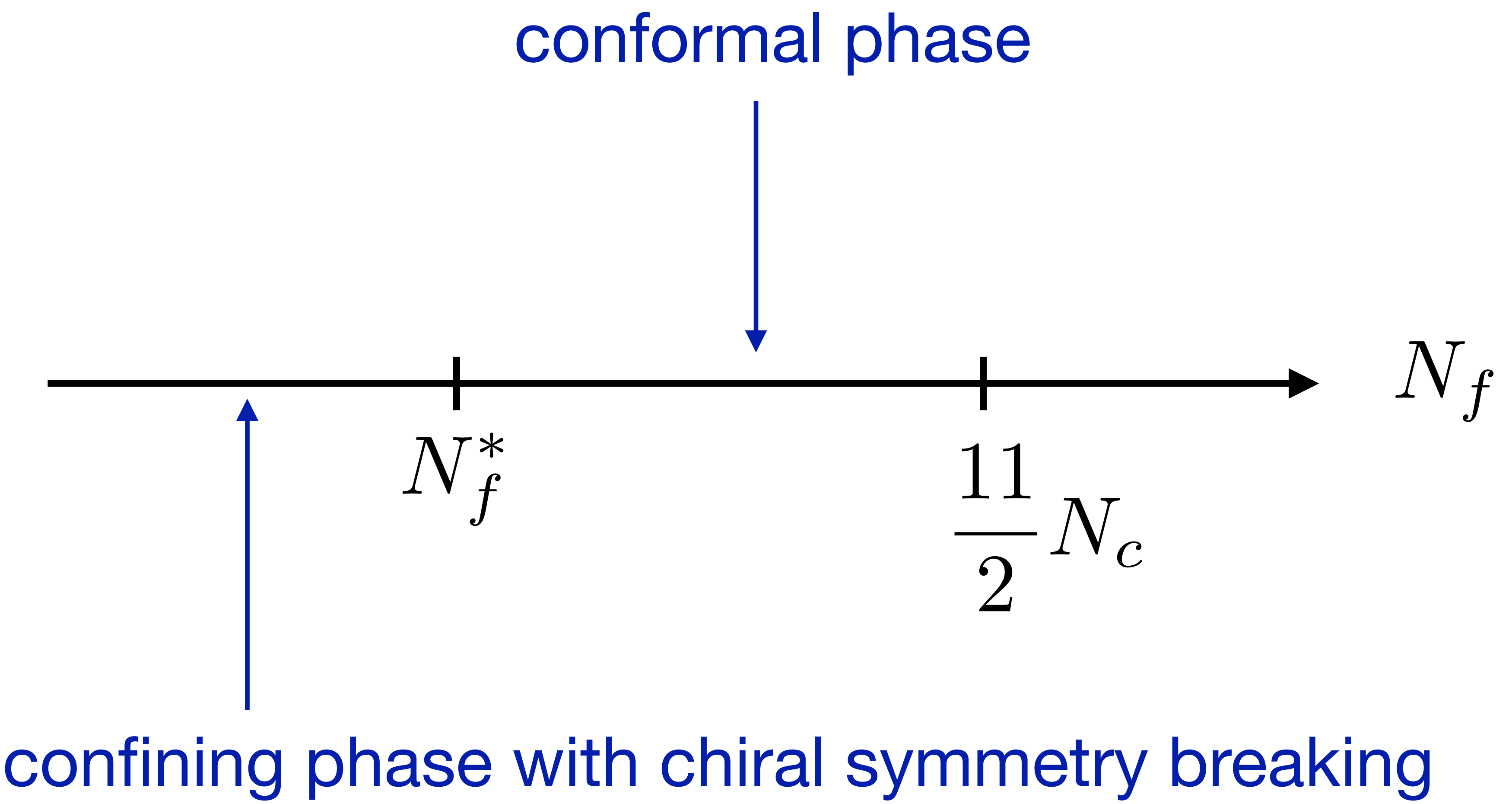
$\frac{N_f}{N_c} = \frac{11}{2} - \epsilon$ => weakly coupled Banks-Zaks fixed point


$(\phi^4)_2$
g/m^2
perturbation theory reliable
Banks-Zaks
N_f
$\frac{11}{2}N_c$

Physics can change qualitatively away from perturbative regime:



Banks-Zaks


conformal phase
N_f
N_f^*
$\frac{11}{2}N_c$
confining phase with chiral symmetry breaking

How can we move beyond perturbation theory?

Resurgence program

Belief: perturbation theory is so rich that it should “know” about nonperturbative physics

How to extract it?

Some perturbative expansions have been shown to be Borel summable (to the exact answer)

E.g. for $(\phi^4)_2$ [Eckmann, Magnen, Seneor 1975]

Can these results be used for practical computations?

Case study: the epsilon-expansion

RG fixed point of ϕ^4 in $4-\epsilon$ dimensions

Critical exponents are computed as power series in ϵ

Divergent but supposedly Borel-summable [Brezin, Le Guillou, Zinn-Justin 1977]

Physically, one is interested in $\epsilon = 1$ (3d Ising model universality class)

$\eta = 0.0365(50)$ after Borel-resumming terms through ϵ^5

[Giuda,Zinn-Justin 1998]

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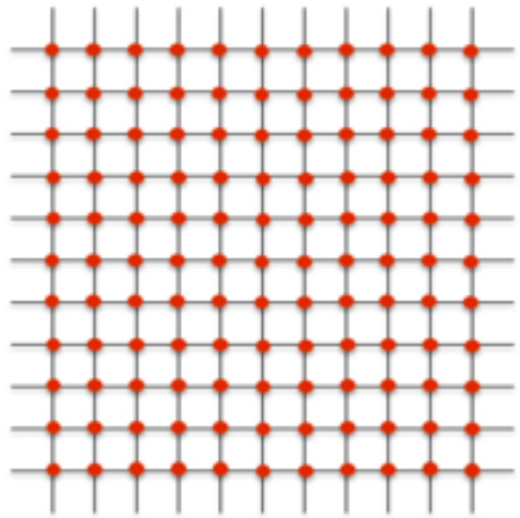
$\eta = 0.0362978(20)$ from conformal bootstrap [Kos, Poland, Simmons-Duffin, Vichi 2016]



Lattice field theory

path integral evaluated on computer

+ works whenever QFT approaches a gaussian fixed point in the UV

+ first principle method

Lattice QCD: tremendously important for establishing the Standard Model and for interpreting precision experiments looking for BSM

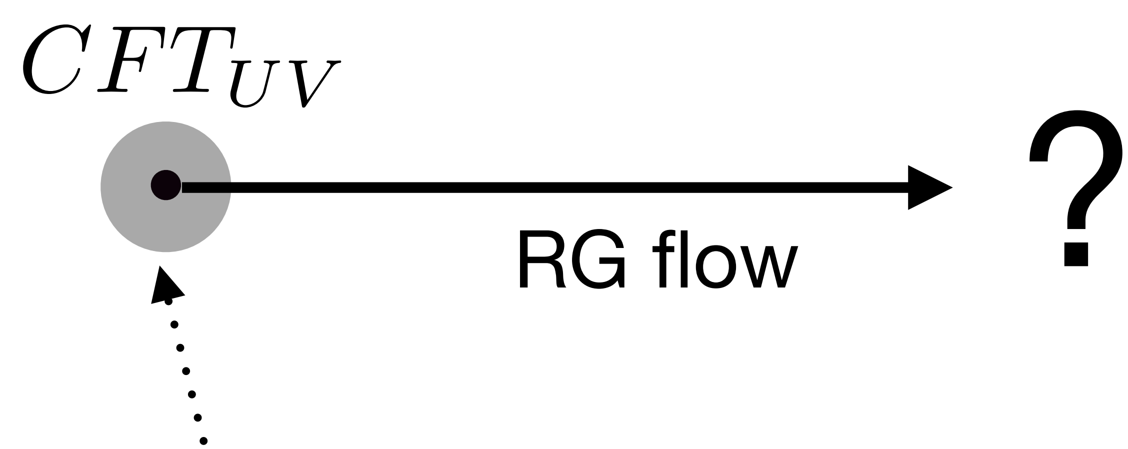
Progress in the field due to human ingenuity as much (if not more) than to computer power increase

- Lattice QCD remains rather expensive. Years of supercomputer time.

Are there any alternative to the lattice worth exploring?

Enlarge the framework

Would also like to study RG flows from non-Gaussian UV fixed points



Is IR theory conformal?
Massive?
Particle spectrum?
S-matrix?

perturbatively makes sense

2 classes of RG flows

1. Perturbation by a relevant operator

$$\Delta S = \mu^{d-\Delta} \int \mathcal{O}_\Delta(x)\, d^d x$$

some CFT operator which is relevant $\Delta < d$
or marginally relevant
(or a linear combination)

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2. Gauging

Suppose UV CFT has a continuous global symmetry
Conserved currents J^a_μ

$$\Delta S = \frac{1}{g^2}\int (F^a_{\mu\nu})^2 + \int J^a_\mu A_a$$

- Relevant for $d \leq 3$
- Marginally relevant in $d = 4$ if nonabelian and $\langle JJ \rangle$ not too large

Can we define such theories nonperturbatively, at least in principle?

Approach 1: first realize CFT on a lattice

Unsatisfactory in practice

Unsatisfactory conceptually

CFTs are defined algebraically

Correlation functions: $\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle$

- Each operator is characterized by its scaling dimension Δ_i

- Operators satisfy OPE algebra (schematically)

$$\mathcal{O}_i \times \mathcal{O}_j = \sum_k \lambda_{ijk} \mathcal{O}_k$$

reduces n-point functions to (n-1)-point functions, converges at finite separation

- CFT data $\Delta_i, \ \lambda_{ijk}$ constrained by OPE associativity

$$(\mathcal{O}_i \mathcal{O}_j) \mathcal{O}_k = \mathcal{O}_i (\mathcal{O}_j \mathcal{O}_k) \qquad \text{(schematically)}$$

CFTs can be defined, studied and constrained via these axioms

Conformal bootstrap program

In 2d [Belavin,Polyakov, Zamolodchikov 1984]

At the time $c<1$ (minimal models).

Presumably vast world of not exactly solvable $c>1$ CFTs could be studied, perhaps numerically, via these axioms.

With natural modifications, these axioms hold for CFTs in $d>2$ and can be used to make concrete predictions about such CFTs [Rattazzi, Rychkov, Tonni, Vichi 2008]

3d Ising model critical point

has been greatly constrained by the conformal bootstrap

$$\eta = 0.0362978(20)$$

$$\nu = 0.629971(4)$$

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi 2012, 2014]
[Kos,Poland,Simmons-Duffin 2014]
[Simmons-Duffin 2015]
[Kos,Poland,Simmons-Duffin, Vichi 2016
[Simmons-Duffin 2016]

NB. Rigorous error bars.
Basically a theorem, assuming the CFT axioms.

Scaling dimensions of about 100 operators and their OPE coefficients are known with some precision (come out of the same computation)

Similar results for other universality classes.

CFTs can be defined and studied algebraically, without recourse to the lattice

=> there must be a way to study RG flows starting from CFTs which only uses CFT data

This would also provide an alternative to the lattice
even when the UV fixed point is gaussian.
Indeed, gaussian massless theories are just particular, simplest, CFTs.

I will now describe one such method.

It is Hamiltonian in nature. We will use the quantum Hamiltonian
to perform spectral computations, approximate but precise.

Recall Rayleigh-Ritz in Quantum Mechanics

$$H = H_0 + V$$

Assume H_0 exactly solvable with discrete spectrum:

$$H_0|n\rangle = E_n|n\rangle$$

View H as an infinite matrix in this basis:

$$H_{mn} = E_n\delta_{mn} + \langle m|V|n\rangle$$

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- Truncate to the first *N* unperturbed energy levels
- Diagonalize truncated matrix on a computer
- Take the limit $N \to \infty$

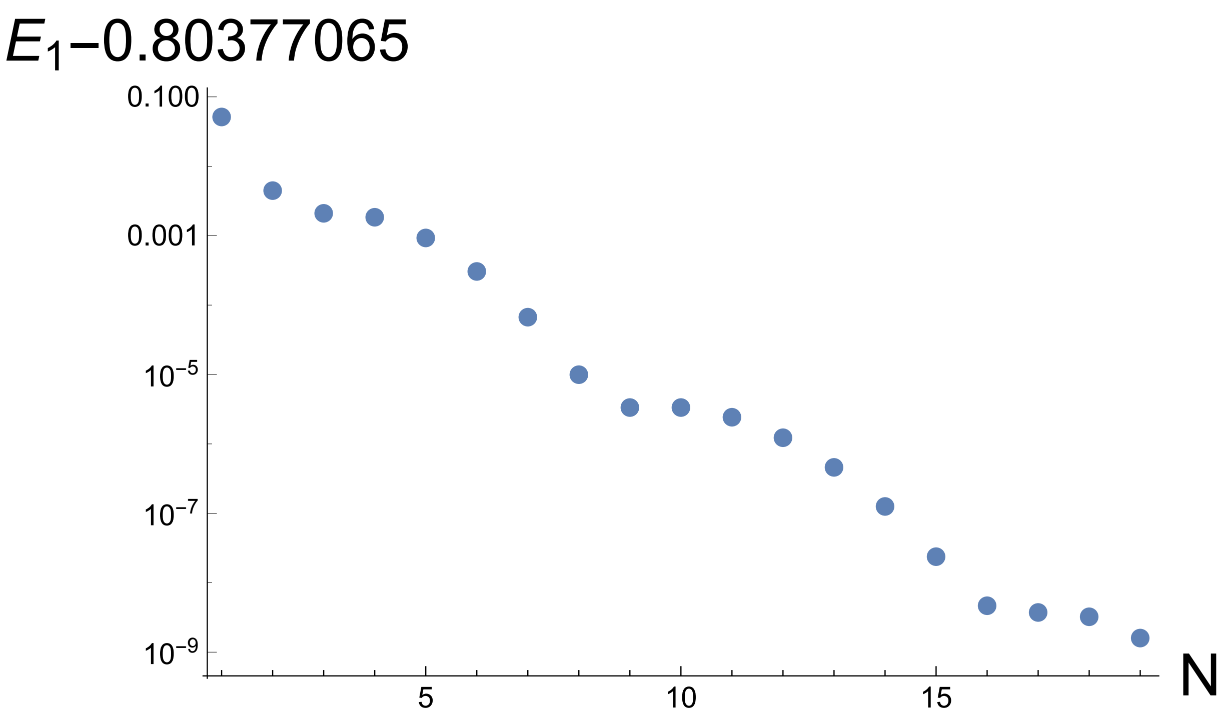
In many cases the limit exists, and reproduces the exact spectrum of *H.*

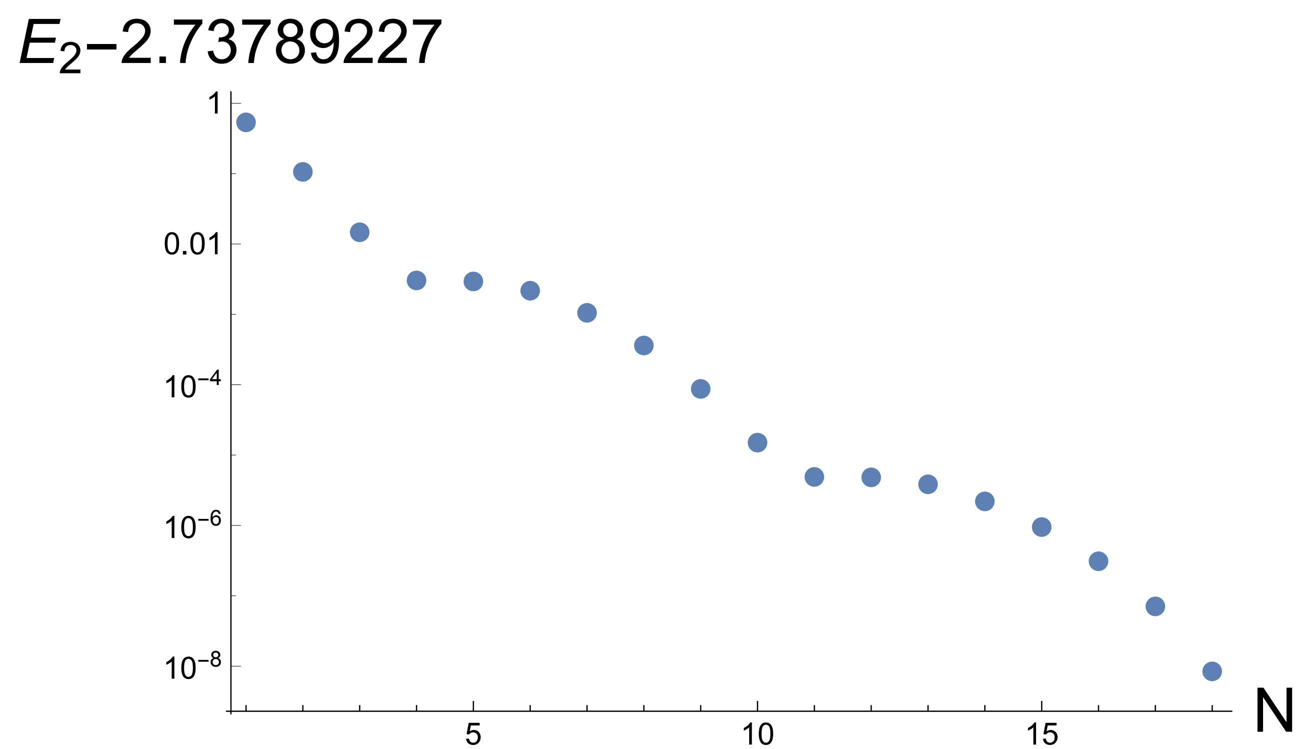
Works even far from the perturbative regime.

E.g. **anharmonic oscillator:**

$$H_0 = \frac{1}{2}p^2 + \frac{1}{2}x^2 \qquad V = \lambda x^4$$

Convergence for the first two eigenvalues $(\lambda = 1)$





Rayleigh-Ritz in Quantum Field Theory

[Brooks, Frautschi 1984]
[Yurov, Al. Zamolodchikov 1990]

The simplest setup: $(\phi^4)_2$

Put in finite volume $0 \le x \le L$ (e.g. periodic) *[L large]*

$$H = H_0 + V$$

H_0 free massive scalar Hamiltonian

$$V = g \int_0^L :\phi(x)^4: \, dx$$

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H_0 free massive scalar Hamiltonian $\qquad V = g \int_0^L :\phi(x)^4: \, dx$

- In finite volume the spectrum of H_0 is discrete
(Fock space of particles with quantized momenta $p_n = \dfrac{2\pi n}{L}$

- Truncate to the subspace of states of the total H_0 energy $\leq E_{\max}$

- Diagonalize truncated H numerically

- Try to take the limit $E_{\max} \to \infty$ (for fixed L)

Does the limit exist?

Results of numerical experimentation:

[Rychkov, Vitale 2014, 2015]
[Elias-Miro, Montull, Riembau 2015]
[Bajnok, Lajer 2015]
[Elias-Miro,Rychkov, Vitale 2017]

+ the spectrum converges

+ convergence rate $\sim 1/E_{\max}^2$

related to ϕ^4 being dimension zero, in general should be $d - 2\Delta_V$

+ can improve to $1/E_{\max}^3$ via a ‘renormalization improvement’

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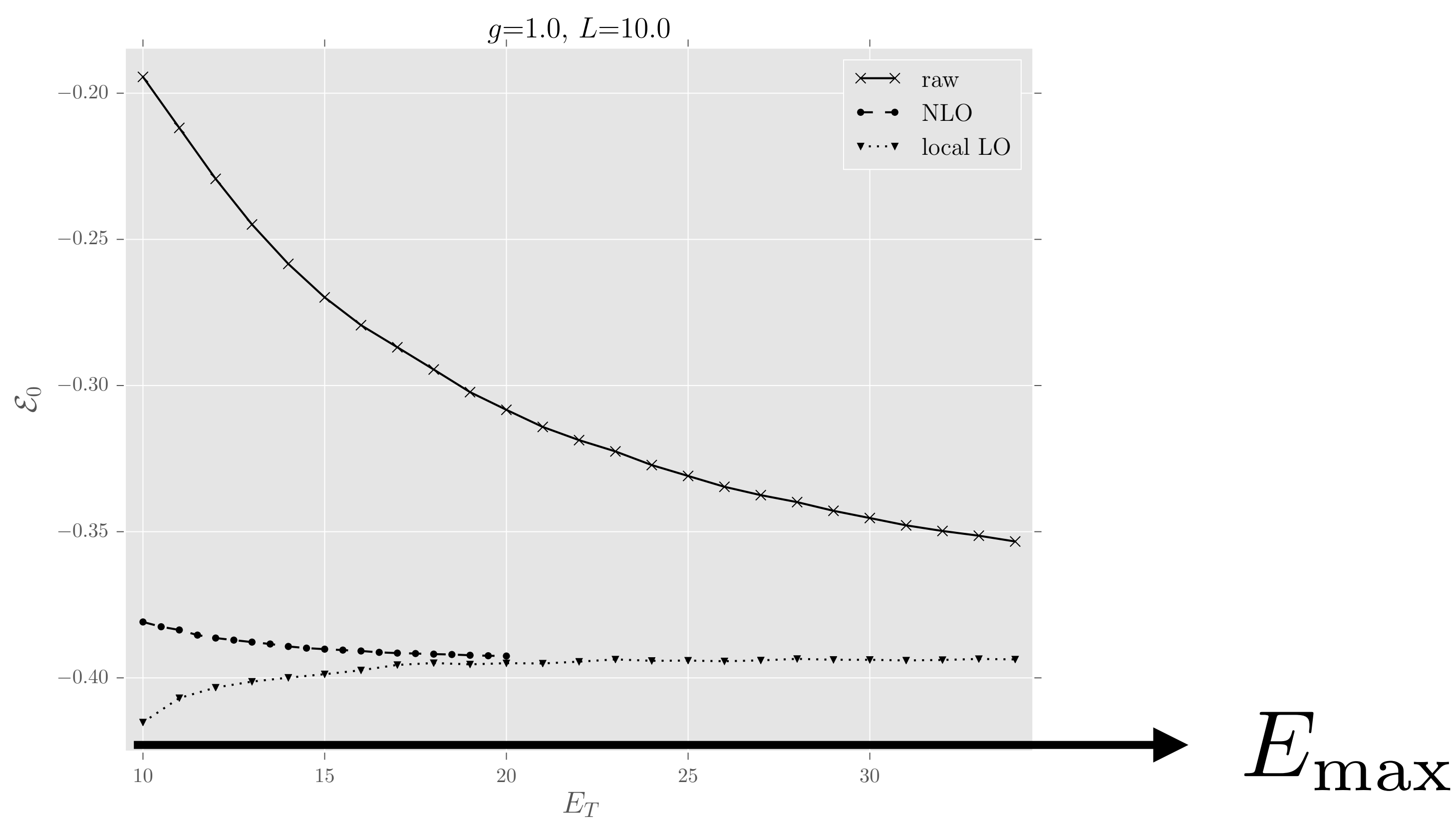
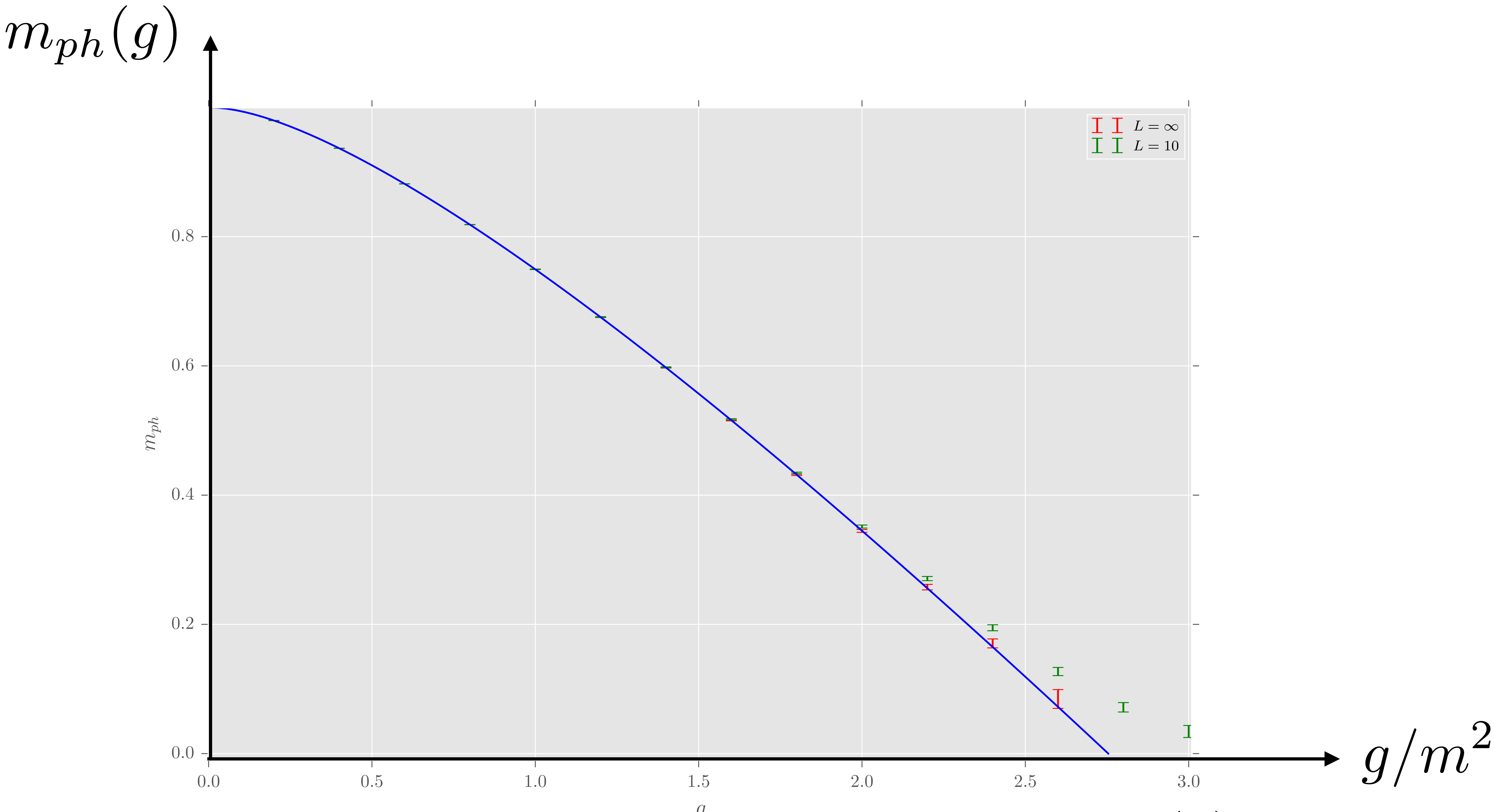
Typical cutoff dependence:



[H.space size, nonlocal cutoff]

Physical mass as a function of the quartic:



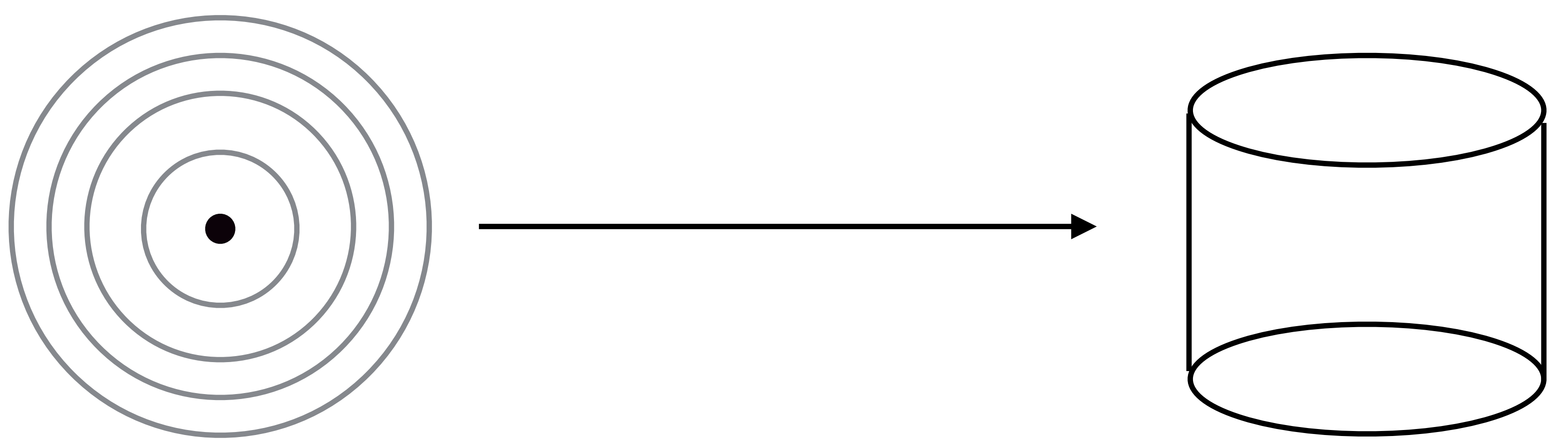
$$g_c = 2.76(3)$$

Hamiltonian method for RG flows starting at CFTs

Any CFT has a canonical way to be put in finite volume

$$\mathbb{R}^d \xrightarrow{\text{Weyl transformation}} \mathbb{R} \times S^{d-1}$$

time x space



"cylinder"

CFT energy levels on S^{d-1} of radius *R* are $\quad E_n = \dfrac{\Delta_n}{R}$

("operator-state correspondence")

Now perturb CFT by $$\mu^{d-\Delta}\int \mathcal{V}\, d^d x$$

On the sphere of radius R we have to study the Hamiltonian:

$$H = H_{\rm CFT} + V$$

$$(H_{\rm CFT})_{mn} = \frac{1}{R}\Delta_n \delta_{mn}$$

$$V_{mn} = \frac{1}{R}(\mu R)^{d-\Delta}\lambda_{\mathcal{O}_m \mathcal{V} \mathcal{O}_n}$$

CFT 3-point function coefficients

To make numerical computations, truncate to CFT states of scaling dimension $\Delta \leq \Delta_{\max}$

"Truncated Conformal Spectrum Approach" (TCSA)

[Yurov, Al. Zamolodchikov 1990]
[Lassig, Mussardo, Cardy 1991]
[Klassen,Melzer 1991]…
recent review: James, Konik, Lecheminant, Robinson, Tsvelik 2017

Most work in d=2, although in principle should work also for d>2

[Hogervorst, Rychkov, van Rees 2014]

- Expected to converge as $\Delta_{\max} \to \infty$ in UV-finite range $\Delta_V < d/2$ beyond which point infinite renormalization is needed.

- Supported by numerics, but would be nice to study carefully