Locality and a bound on entanglement assistance to classical communication

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(work in progress; joint with **P.E. Frenkel**)

Quantum Information and Operator Algebras
Rome, 16 february 2018

2 headed oracles

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$\Rightarrow$ all 2-headed oracles form a polytope

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- Quantum: $\varphi(A_{x_1,z_1} B_{x_2,z_2})$ where
  - φ is a positive normalized functional
  - $A_{x_1,z_1} \geq 0, \sum_{z_1} A_{x_1,z_1} = I$ & sim. cond. for B
  - $[A_{x_1,z_1}, B_{x_2,z_2}] = 0$

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$\rightsquigarrow$ "NS-oracle" $\in$ "No-signaling polytope"

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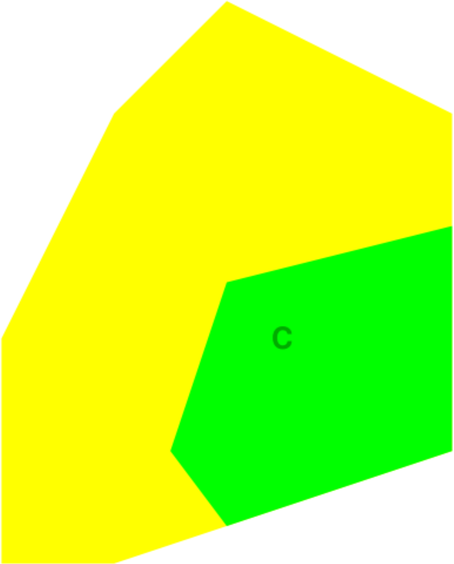

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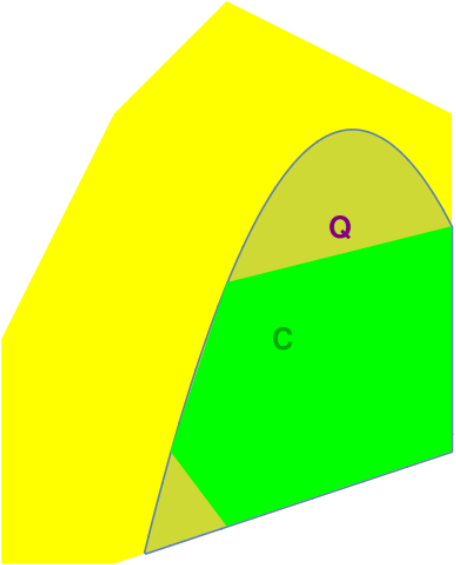

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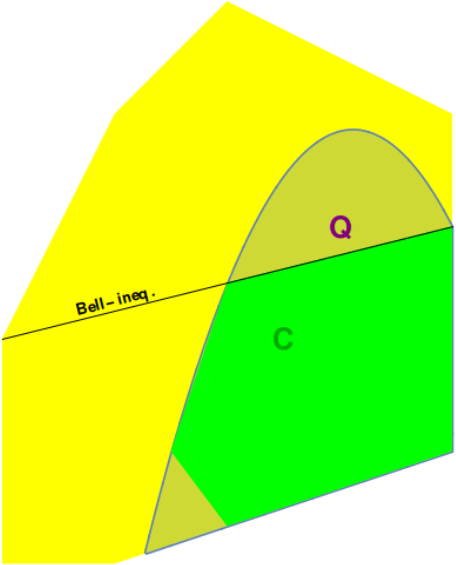

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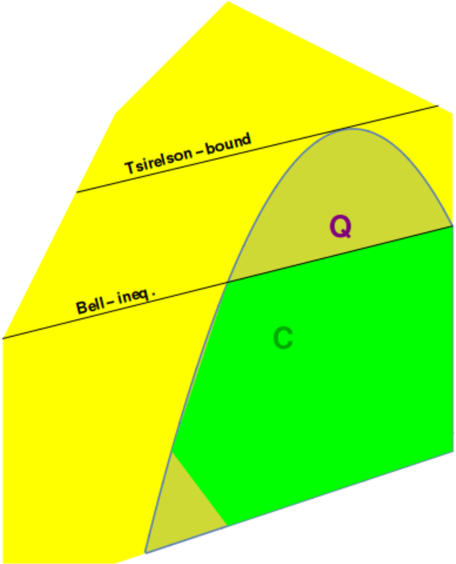

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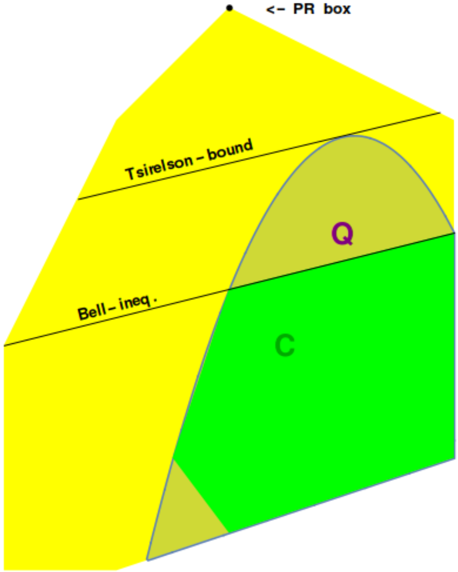

Information Causality

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⇓

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$\Downarrow$

$\Rightarrow$

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$\Rightarrow \quad \Rightarrow$

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$$\sum_{k=1}^{n} I(d_k : g \,|r = k)$$

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⇓ ⇒ ⇒ ⇒ ⇒ $\underline{h}$: m cbits help ⇒ ⇒ ⇒ ⇒ ⇑

$$\sum_{k=1}^{n} I(d_k : g \,|r = k) \leq m$$

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- implies the Tsirelson-bound

Task: information sending

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X

$\Downarrow$

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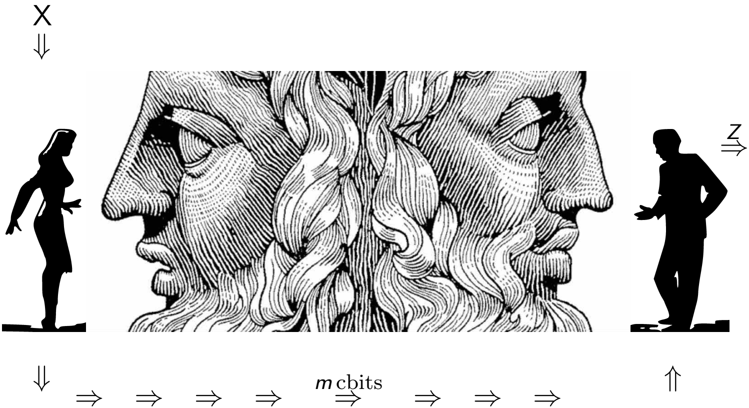

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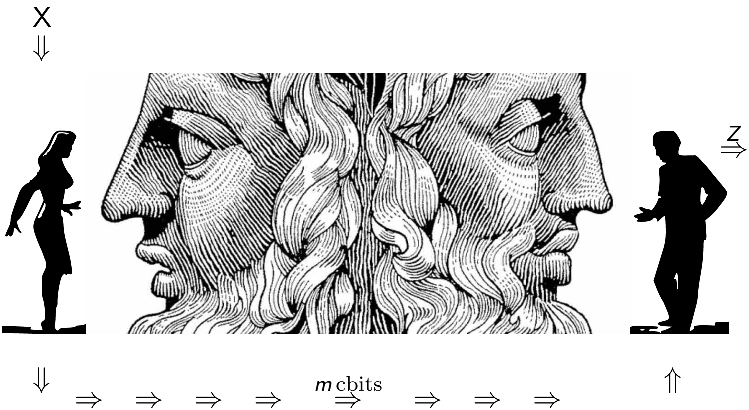

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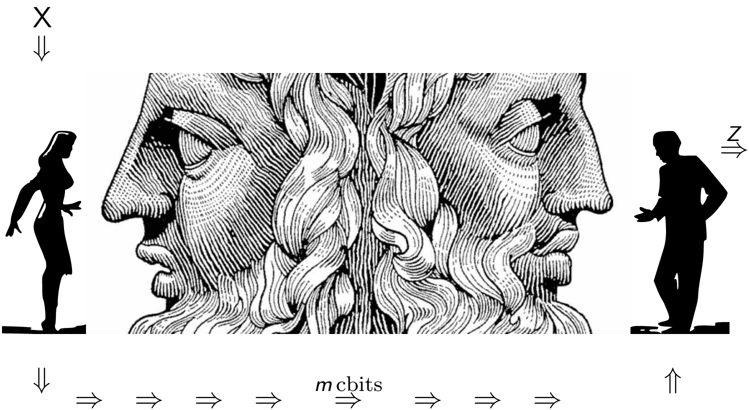

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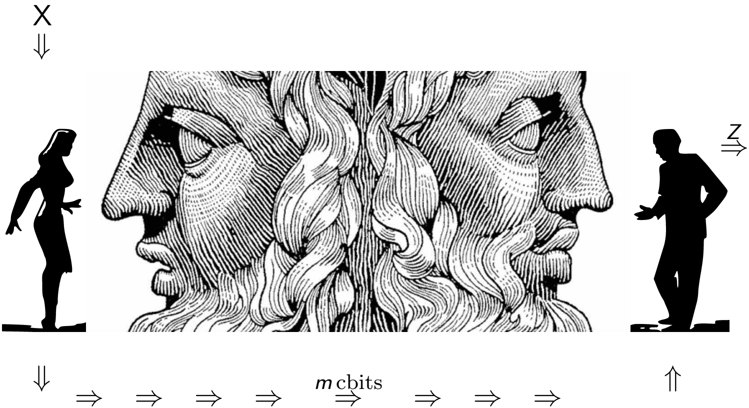

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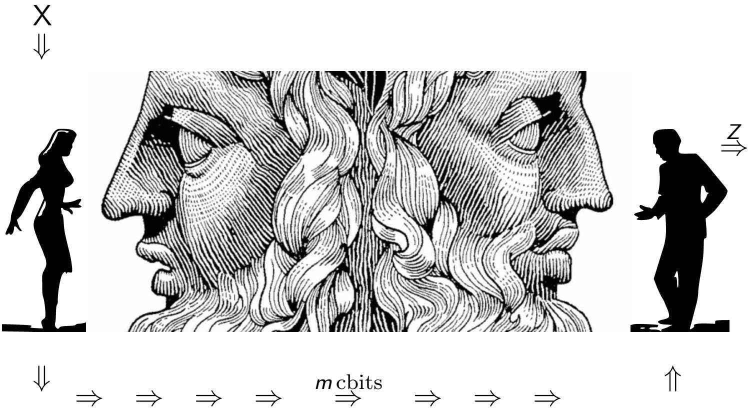

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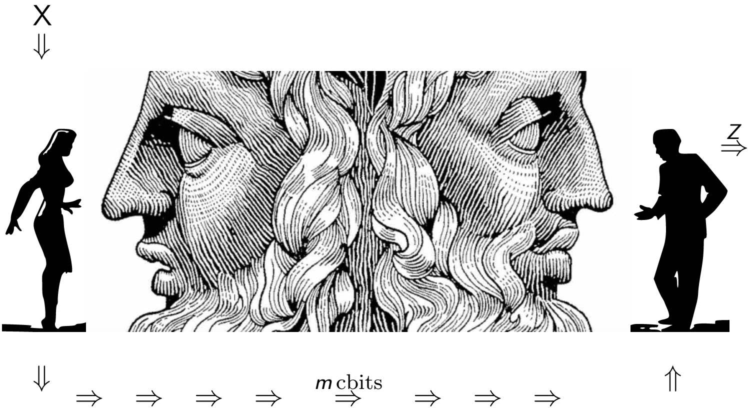

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with a PR-box, $p(\mathrm{win}) = 1$ is achievable!

Exact classical simulation

Definition

$C_{j,k}(t)$: set of all $j \times k$ channel matrices realizable by transmitting a classical t-level system (that is, $m = \log_2(t)$ cbits) and using a common source of randomness

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In a game with 3 inputs / outputs, ns-oracles can be of any help

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state space $\rightsquigarrow$ a convex body K; e.g. classical simplices

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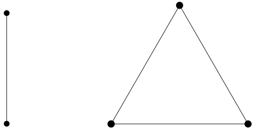

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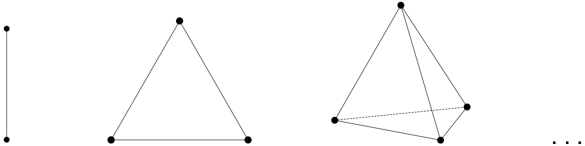

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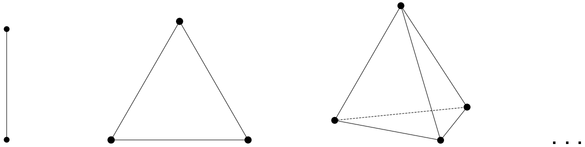

quantum case: $S_1^+(\mathbb{C}^d)$

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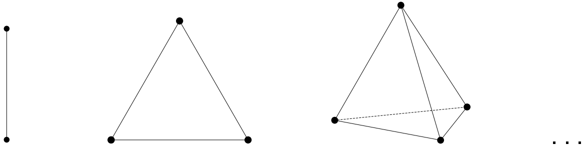

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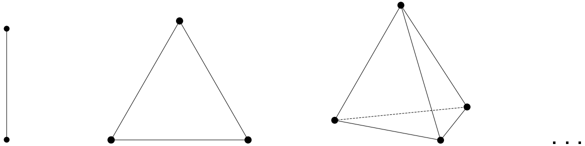

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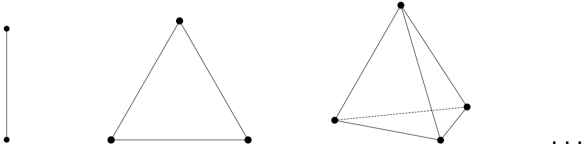

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Question:

what is the minimum t value for which $C_{j,k}(t)$ contains all $j \times k$ channel matrices realizable by transmitting a single isolated system whose state space is K?

Classical capacity of convex bodies

In general, $C_{j,k}(t)$ is not characterized by the "trivial" bounds and the fact that all of its elements have channel capacity $\leq \log_2(t)$

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Answer ☹

A lot of other bodies would be still ok; nothing specific about the 3-dim ball. E.g. $\square$ is also a "1-bit space".

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- $p(z_1, z_2|x_1, x_2) = \Phi^{x_2}_{z_2}(q^{x_1}_{z_1})$

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- $x_1 \to z_1$ measurement on I $\rightsquigarrow q$ "changes" to $q_{z_1}^{x_1} \in K$
- ns-condition: $\sum_{z_1} p(z_1|x_1) q_{z_1}^{x_1} = q$
- x_2 measurement on II: $\Phi^{x_2} : K \stackrel{affine}{\to}$ simplex
- $p(z_1, z_2|x_1, x_2) = \Phi_{z_2}^{x_2}(q_{z_1}^{x_1})$

Conclusion

Considering all "initial states" $q \in K$ together with every convex decomposition of it + every possible $K \stackrel{affine}{\to}$ simplex map we can construct the set of all possible ns-oracles arising from bipartite physical systems where one part has state space K.

From one part to bipartite

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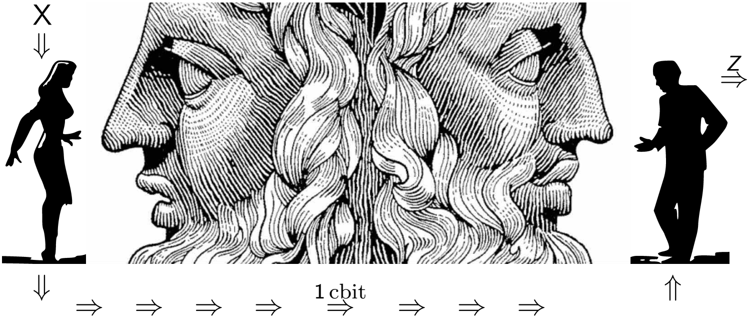
$\hookrightarrow$ Further restrictions on what the state space of a 1-bit system can be. E.g. $K = \square$ would allow realization of the PR-box.

Nontrivial bounds: results



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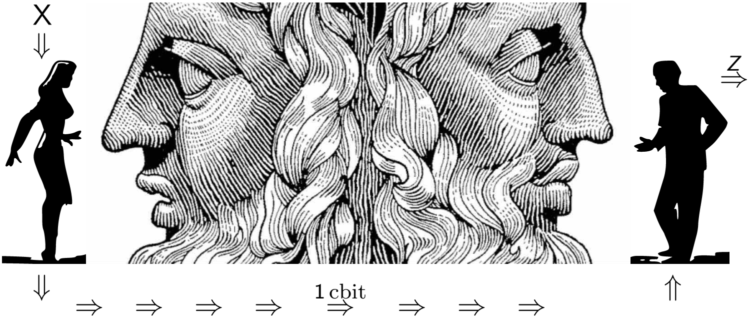

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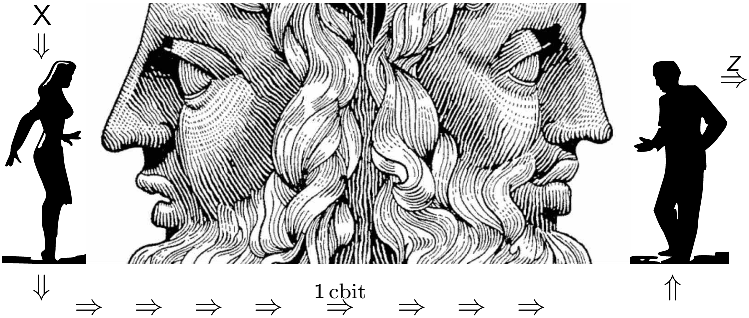

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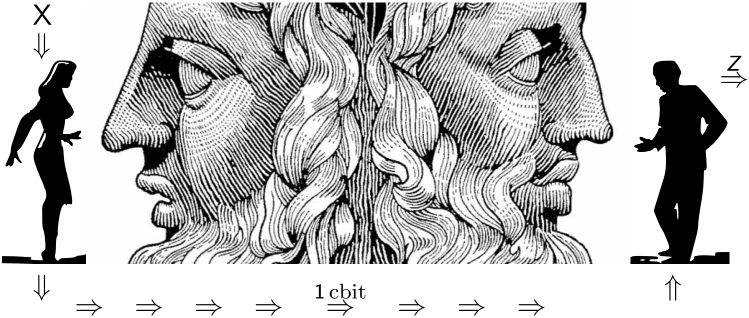

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- $\forall n \; \exists$ example with some ns-oracle that **cannot** be simulated classically even if we allow n cbits to be sent instead of 1
- Possible principle? "*God helps those who help themselves.*"