THE MESSAGE OF QUANTUM MECHANICS

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"If someone tells you they understand quantum mechanics then all you've learned is that you've met a liar."

(R.P. Feynman)

"Anyone who is not shocked by quantum theory has not understood it."

(N. Bohr)

"We have to ask what it means!"

(K.G. Wilson)

Contents

Credits

Coleman, Fierz, Hepp;
Dürr; Gell-Mann & Hartle;
Goldstein; Griffiths; Omnès;
Pickl, Schilling, ...

1. Introduction

20th Century Physics has brought us

- vindication of an old paradigm: "atomism" k_B

&

- three revolutions:

  Quantum Mechanics $\hbar$

  STR c^{-1}

  GR ℓ_P

$c^{-1}, \ell_P \Rightarrow$ loss of predictability of future; observer-dependence

$k_B, \hbar \Rightarrow$ loss of determinism & "realism".

New theories arise as "deformations" of precursor ths.

$k_B, \hbar$: Deformations of assoc. abelian alg. (of fns. over phase space)

Examples!

c^{-1} : Deformation of symm.

l_P : ——"—— of geometry

More recent progress: Combi. of ≤ 3 out of $k_B, \hbar, c^{-1}, l_P$.

Program: In which way do new theories differ from precursor "class." theories; how do the latter reappear in limiting regimes of the former?

Today: k_B & $\hbar$

Is atomistic QM a realistic theory that tells us what happens – rather than just what might happen?

Rôle of notions such as "event", "observer", "frequency"

Original pt. of view of Schrödinger: $[\psi_t]$ tells us "what happens". Schrödinger eq. – lin. or NL – = Hamiltonian evolution eq. for wave field $\psi_t(x)$.

This int. of QM is not tenable (Heisenberg, Dirac, Born) – unstable against def. ("2nd quantization) → many-body QM, w. non-abelian alg. of "obs."

2. What is a physical system? Realistic vs. "Idealistic" Theories

Phys. system, S, specified in terms of observable physical quantities rep. as lin. operators;
→ generate *alg. $\mathcal{A}_S \subseteq \mathcal{B}_S$;
$\mathcal{B}_S$: algebra of "possible events", (given some exp. equipment, $\mathcal{O}$).

Fundamental data:

(I) $\mathcal{A}_S \subseteq \mathcal{B}_S$: a C^*-alg. (dep. on $\mathcal{O}$)
(II) $\mathcal{S}_S$: "states" on $\mathcal{B}_S$ (standard)
(III) G_S: "symmetries" of S, incl. time evolution.

Symm. trsfs. from G_S act as *automorphisms on $\mathcal{B}_S$; ex.: time evolution: $(t,s) \mapsto \alpha_{t,s} \in \mathrm{Aut}(\mathcal{B}_S)$.

(IV) Subsyst./composition:

$$S \subset S' \Rightarrow \mathcal{B}_S \subset \mathcal{B}_{S'}$$

$(S_1, S_2) \mapsto S_1 \vee S_2$, with $\mathcal{B}_{S_1 \vee S_2} = \mathcal{B}_{S_1} \otimes \mathcal{B}_{S_2}$

If $S_1 \simeq S_2 \simeq S$ specify imbedding:

$\mathcal{S}_{S \vee S} \hookrightarrow \mathcal{S}_S \otimes \mathcal{S}_S$: statistics

Choice of (I), (II), (III) depends on equipment available to observe Nature, $\mathcal{O}$ ("observer").

New theories arise by "deformations" of (I), (III), (IV); (Flato, Faddeev)

(I) [cont. ths. of matter $\xrightarrow{k}$ atomism

class. mechanics $\xrightarrow{h}$ QM

(III) Galilei symm. $\xrightarrow{c^{-1}}$ Poincaré $\xrightarrow{R^{-1}}$ de Sitter

(IV) [permutation stat. $\rightarrow$ braid stat.

group symm. $\rightarrow$ quantum groups

$\updownarrow$

th. of braided $\otimes$ categories, duality (Tannaka-Krein th.)

<u>Ex.</u> Vlasov th. $\xrightarrow{k}$ Newtonian mech.

$\downarrow h$

wave mechanics

realistic ("class.") theories R

"idealistic" (quantum) ths. Q

(R) Realistic theories

- $\mathcal{B}_S$ abelian $\overset{\text{Gel'fand}}{\Longrightarrow}$ $\mathcal{B}_S \simeq C_0(M_S)$

$$M_S = \operatorname{spec} \mathcal{B}_S \quad (\text{e.g., } M_S = \Gamma)$$

- $\mathcal{S}_S = \{\text{prob. measures on } M_S\}$

Pure States $= \{\delta\text{-fns. on } M_S\}$

$\updownarrow \qquad \simeq \{\text{chars. of } \mathcal{B}_S\}$

no superposition principle;

no entanglement betw. S_1 & S_2 in $S_1 \vee S_2$.

- *automorphisms of $\mathcal{B}_S$

$\overset{1-1}{\longleftrightarrow}$ homeomorphisms of M_S

Problem 1. When does TM_S exist (is M_S a diff., (sympl. $\cdots$) mf.)? If it does then time evol. $\{\alpha_{t,s}\}$ generated by VF, X_t, on M_S:

$$\dot{\xi}_t = X_t(\xi_t), \quad \xi_t \in M_S.$$

→ Realistic & det. descr. of S!

$$P_i := \alpha_{t_i,t_0}(\chi_{\Omega_i}) = \chi_{\Omega_i} \circ \phi_{t_i,t_0} = \chi_{\phi^{-1}_{t_i,t_0}(\Omega_i)}$$

Then, for $\xi_0 \in M_S$,

$$\delta_{\xi_0}\left(\prod_{i=1}^{n} P_i\right) = 0 \text{ or } 1 \, !$$

"Effective" dynamics:

$$T_{t,s} : \mathcal{S}_S \to \mathcal{S}_S, \quad \text{w. } T_{t,s} \circ T_{s,u} = T_{t,u}$$

→ Stoch. processes on M_S!

(Q) Quantum theories

$\mathcal{A}_S$, hence $\mathcal{B}_S$, non-abelian

<u>Example</u>: $\mathcal{B}_S$ type-I C^*-alg.

(e.g. group alg. of compact Lie group – qm spins – or Weyl for S w. finite nb. of degs. of freedom)

$\mathcal{Z}_S :=$ centre of $\mathcal{B}_S$: abelian $\to$ R

- $\mathcal{B}_S \simeq \int^{\oplus}_{\mathrm{spec}\,\mathcal{Z}_S} B(\mathcal{H}_\xi)$, $\xi \in \mathrm{spec}\,\mathcal{Z}_S$,

$\mathcal{H}_\xi$: Hilbert space

$\mathcal{S}_S = \{\text{density matrices on } \mathcal{H}.\} \otimes \{\text{prob. meas. on spec } Z_S\}$

Pure states

$= \{\text{unit rays in } \mathcal{H}_\xi \mid \xi \in \text{spec } Z_S\}$

- Superposition princ. within every $\mathcal{H}_\xi$
- Entanglement betw. S_1 & S_2 in $S_1 \vee S_2$

$\mathcal{A}_S$ non-commutative

uncertainty rels. (Heisenberg)

decoh. (S "cat") entanglement (Schrödinger)

Bell <, CHSH <, (Kochen-)Specker

Tsirelson ← Grothendieck

Theory intr. probabilistic

→ sep. lect.

Digression

Problem 2. (Quantum marginal pr. ↗ A. Klyachko)

$\mathcal{B}_{S_i} \simeq B(\mathcal{H}_i)$, $\dim \mathcal{H}_i < \infty$, $i = 1, \cdots, n$

Then $\mathcal{B}_{S_1 \vee \cdots \vee S_n} = \bigotimes_1^n B(\mathcal{H}_i)$.

Let P be a density matrix on $\mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n$, P_i its i^{th} "marginal", i.e.,

$$tr(P\, \mathbb{1} \otimes \cdots \otimes \underset{\substack{\uparrow \\ i^{th}\text{ slot}}}{a} \otimes \cdots \otimes \mathbb{1}) =: tr(P_i a)$$

$\forall a \in B(\mathcal{H}_i)$.

Find conds. on $P_1, \cdots, P_n$

implying that $\exists\, P$ s.t.

$P_i = i^{th}$ marginal of P, $i = 1, \cdots, n$.

($\nearrow$ A. Klyachko, ...)

Note: P pure $\Leftrightarrow$ $P = P_\Psi$, $\Psi \in \mathcal{H}$.

S_1, S_2 entangled in $P = P_\Psi$,

$\Psi \in \mathcal{H}_1 \otimes \mathcal{H}_2 \Leftrightarrow P_1, P_2$ not pure.

<u>Exc.</u> $P = P_\Psi \Leftrightarrow P_1$ & P_2 are isospectral

($\nearrow$ Schmidt dec.)

Some known results (on demand).

<u>Problem 3.</u> Similar problems w.

$S_i \simeq S$, $i = 1, \cdots, n$, $\mathcal{H}_{S_1 \vee \cdots \vee S_n} = \bigotimes_{i=1}^{n\,\Delta} \mathcal{H}_{S_i}$:

rôle of statistics! DFT.

Further implications of non-commutativity of $\mathcal{A}_S$:

(A) Uncertainty relations

(B) Non-existence of hidden variables:

$$\mathcal{A}_S \ni a = a^* \mapsto \alpha_a : \Omega \to \mathbb{R} \quad (RV)$$

$$\mathcal{S} \ni P_\Psi \mapsto \rho_\Psi \quad (\text{prob. meas. on } \Omega)$$

with

(i) $\langle \Psi, P_a(\Delta) \Psi \rangle = \rho_\Psi(\alpha_a^{-1}(\Delta))$, $\forall \Delta \subset \mathbb{R}$; and

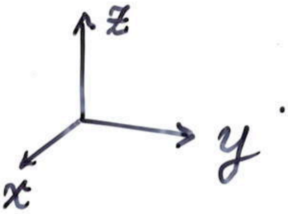
(ii) $\alpha_{f(a)} = f(\alpha_a)$, for arb. continuous f on $\mathbb{R}$.

Theorem. (Kochen-Specker)

If $\mathcal{A}_S \supseteq B(\mathcal{H})$, $\dim \mathcal{H} \geqslant 3$ then embedding satisfying (i) & (ii) is impossible.

<u>"Exc."</u> Spin 1. Then S_x^2, S_y^2, S_z^2 are comm. orth. proj. with

$$S_x^2 + S_y^2 + S_z^2 = 2 \cdot \mathbb{1}_{3\times 3},$$

for an arb. choice of .

If K-S map existed then $\exists$ fu., σ, on unit sphere, S^2, s.t.

$\sigma(\vec{n}_i) = 0$ or 1, $i = 1, 2, 3$,

$\sum_1^3 \sigma(\vec{n}_i) = 2$, for an arb. orth. "3-bein" $(\vec{n}_1, \vec{n}_2, \vec{n}_3)$.

Such a fu. σ does not exist.

(drawing!)

(C) Correlation matrices & Bell <'s

$S = S_1 \vee S_2$, $\mathcal{A}_{S_i} = B(\mathcal{H}_i)$, $i = 1,2$.

P : state of S; $a_1, \dots, a_K$ in $\mathcal{A}_{S_1}$, $b_1, \dots, b_L$ in $\mathcal{A}_{S_2}$ satisfying

$$a_k^2 = 1 \ , \ b_\ell^2 = 1 \quad \forall k, \ell.$$

$$\Gamma_{k\ell} := tr(P a_k b_\ell) \qquad (Q)$$

All such Γ's form a compact convex subset, M_Q, in $M_{K \times L}(\mathbb{R})$.

If all a_k's and all b_ℓ's commute then (spect. thm.):

$$\Gamma_{k\ell} = \int_\Omega \alpha_k(\omega)\, \beta_\ell(\omega)\, d\rho(\omega), \quad (C)$$

with $\alpha_k^2 = \beta_\ell^2 \equiv 1$, $\forall k, \ell$, where $d\rho$ is a prob. measure.

Matrices as in (C) form a compact conv. subset, $\mathcal{M}_C$, in $M_{K\times L}(\mathbb{R})$.

Obviously $\mathcal{M}_C \subseteq \mathcal{M}_Q$.

<u>Theorem.</u> (B. Tsirelson)

$$\Gamma \in \mathcal{M}_Q \Rightarrow K_G^{-1}\Gamma \in \mathcal{M}_C,$$

$K_G \approx 1.73 \pm 0.06$ (Grothendieck's cst.)

(exact when K, L arb.; for $K = L = 2$, $K_G \to \sqrt{2}$).

$$\mathcal{M}_C \subsetneqq \mathcal{M}_Q$$

<u>Example</u> (exc.): CHSH <

$$\overline{\Gamma} := tr\left(P[a_1(b_1+b_2)+a_2(b_1-b_2)]\right)$$

(i) $\max\limits_{P,\underline{a},\underline{b}} \overline{\Gamma} = 2\sqrt{2}$

(ii) If $\Gamma \in \mathcal{M}_C$ then $\overline{\Gamma} \leq 2$

Grothendieck' Constant

$$K = (k_{ij})_{i,j=1}^{n}, \quad n \geq 2, \quad s.t.$$

$$\left| \sum_{i,j=1}^{n} k_{ij} s_i t_j \right| \leq 1,$$

$\forall \; \underline{s} = (s_1, \cdots, s_n), \; |s_i| \leq 1, \; i = 1, \cdots, n,$

$\forall \; \underline{t} = (t_1, \cdots, t_n), \; |t_j| \leq 1, \; j = 1, \cdots, n.$

Then $\exists$ constant K_G s.t.

$$\left| \sum_{i,j=1}^{n} k_{ij} \vec{\sigma}_i \cdot \vec{\tau}_j \right| \leq K_G(n)$$

where $\vec{\sigma}_i, \vec{\tau}_j \in \mathbb{E}^M$ (M arb.) with

$|\vec{\sigma}_i| \leq 1, \; |\vec{\tau}_j| \leq 1.$

$$K_G(2) = \sqrt{2}, \quad K_G(3) < 1.517, \quad K_G(4) \leq \frac{\pi}{2}$$

$$\lim_{n \to \infty} K_G(n) =: K_G \stackrel{?}{=} \frac{\pi}{2 \ln(1+\sqrt{2})}$$

(no realistic int. of QM ↗ 3; non-extendability - Colbeck & Renner)

- Dynamics $\{\alpha_{t,s}\}$ → unitary propagators on $\mathcal{H}$. × flow on spec Z_S.

Eff. dynamics: "Lindbladians"

Preparation: S can be "prepared" in pure states $[\psi]$, $\psi \in \mathcal{H}_\xi$, without further info. on initial conditions. In exp., observable $a = a^* \in \mathcal{A}_S$ can be measured with $[a, P_\psi] \neq 0$.

"Attracting families of states" → separate lecture!

3. Why must a "realistic" interpretation of QM fail? The no-signaling lemma

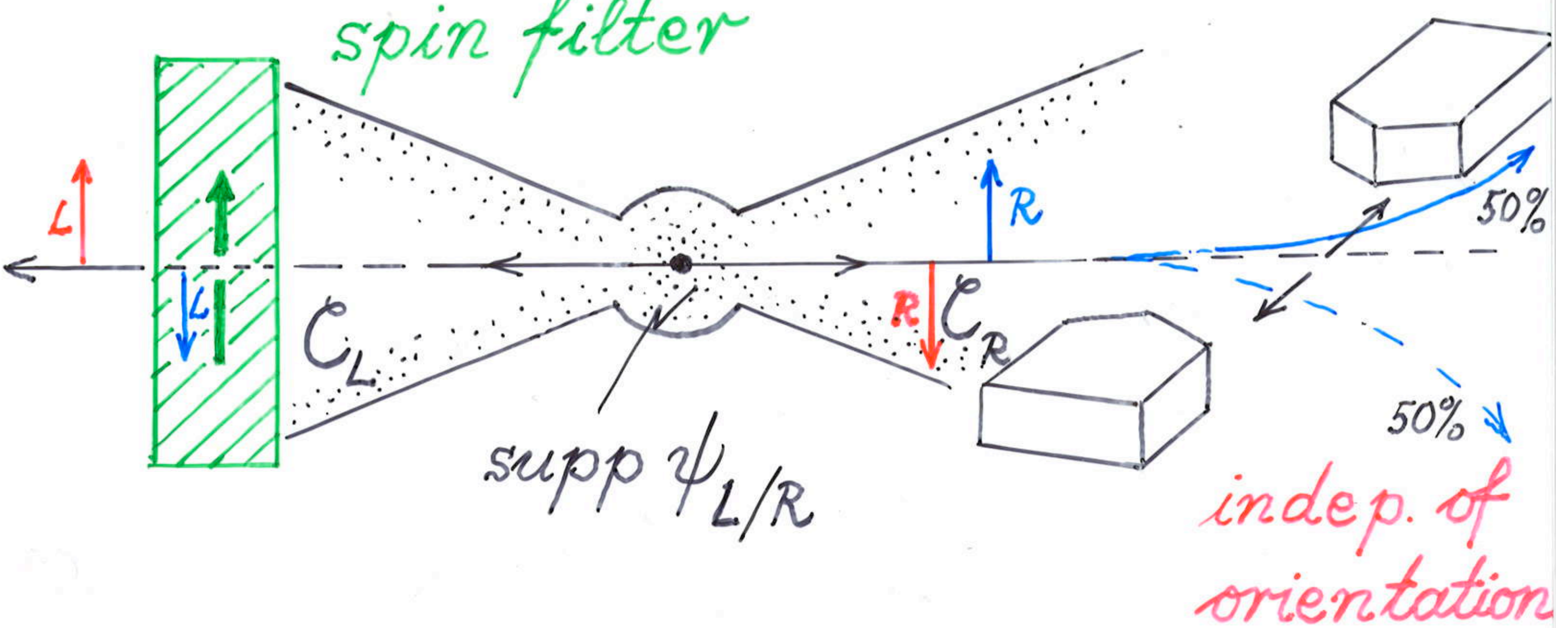
Realistic (det.) int. of QM: ...

QM does sometimes predict facts: ... (Theory of preparation of (sub-) systems)

Yet, there remains an irred. element of chance whenever $\mathcal{A}_S$ is non-abelian.

Example (no-signaling lemma – F-P-S) 2 electrons prep. in spin singlet state:

$$\Psi^{(2)}_{\cdot} := (\psi_L \otimes \psi_R + \psi_R \otimes \psi_L) \otimes (|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle)$$



$L\uparrow$ transm., $\downarrow L$ absorbed

Experiment: If $L\uparrow$ observed then $R\downarrow$ predicted; (1)

if $\downarrow L$ obs. then $\uparrow R$ predicted

$\psi_{L/R}$ propagates into $\mathcal{C}_{L/R}$, except for tiny tails.

Initial state of composed syst.:

$$\Phi_0 = \sum_\alpha \Psi^{(2)}_\alpha \otimes \chi_{\text{filter},\alpha}$$

Dynamics: $H = H_0 + H_I$

H_0 : dyn. of uncoupled syst.

H_I : int. electrons-filter localized around filter.

$$\Phi_t := e^{-itH}\Phi_0$$

<u>Lemma</u> ("no signaling")

Under "reasonable hyp." on H_I,

$$\langle \Phi_t, \vec{S}_{\mathcal{C}_R}\Phi_t \rangle \simeq 0, \qquad (2)$$

for all t.

Consequ. of Cook arg. & "cluster props." – Suppose that (R)

$$\langle \Phi_t, \vec{S}_{\mathcal{C}_L}\Phi_t \rangle = (\hbar/2)\,\vec{e}_3 \qquad (L\uparrow)^*$$

(2) & $(L\uparrow)^*$, or $(\downarrow L)$, contradict (1)!

$\Rightarrow$ $(L\uparrow)^*$, i.e. (R), impossible!

$\Rightarrow$ Frequ. of transm. of left el. through filter $\simeq 1/2$, …

Thus, Φ_t does not describe what happens in a det. way, but only what may happen.

Einstein causality not invoked.

$\nearrow$ J.F., P.P., C.S.

4. Some fundamental notions and questions about QM

System, S, (e.g., e^-, atom, ...) to be explored w. exp. equipment, E, (e.g., lasers, magnets, detectors, filters, ...).

$S \vee E$ described by ($\nearrow$ 2, (I)):

(I) C^*-alg. $\mathcal{B} \equiv \mathcal{B}_{S \vee E}$, "states" $\mathcal{S}\,(\ni \omega, \rho, \cdots)$ on $\mathcal{B}$

(II) Time evolution, $\{\alpha_{t,s}\}_{t,s \in \mathbb{R}}$, in Heisenberg picture (*automorphisms of $\mathcal{B}$)

(III) $\mathcal{B} \supset \mathcal{A}_S$: v. Neumann alg.

generated by operators repr. <u>phys. quantities</u> referring to S at some time t_0, (comp. of position, momentum, spin of particle belonging to $S \cdots$, at time t_0).

For obs. $a = a^* \in \mathcal{A}_S$,

$$a_t := \alpha_{t,t_0}(a) \in \mathcal{B}$$

denotes obs. corresp. to a at time t.

$P_{a_t}(I)$: spect. proj. of a_t corresp. to $I \subset \mathbb{R}$.

$$P_{a_t}(I)^{\perp} := P_{a_t}(I^c) = \mathbb{1} - P_{a_t}(I)$$

Questions to be raised:

(1) How can one prepare spec. initial states, ω, of $S \vee E$ at time t_0 when exps. start? (Th. of prep. $\leftrightarrow$ attracting fams. of states $\leftrightarrow$ relaxation to meta-stable states & g.s.: sep. lect.)

(2) Exps. on S, using E, done to produce "obj. events/facts", e.g., measured values of obs. $a_t = a_t^*$, $a \in \mathcal{A}_S$. – What are "possible events" in QM?

"Possible event" at time t $\leftrightarrow$ spect. proj. $P \equiv P_{a_t}(I)$, $a \in \mathcal{A}_S$.

"Ontology" underlying QM:

Time-ordered sequs. of "events" = "histories" triggered by E.

E, $\{\alpha_{t,s}\}$ chosen such that a class of events $\{P_{a_t}(I)\}_{I \subset \mathbb{R}} \leftrightarrow$ "super sel. sectors" of E (~"pointer positions") identifiable in non-demolition measurements of E.

$\Rightarrow$ No ∞ sequ. $E \leftarrow E' \leftarrow E'' \leftarrow \cdots$ needed to trigger events!

(3) Rôle of E in rendering possible events $P \equiv P_{a_t}(I)$ & $P^{\perp}$ complementary, given sub-

sequent events (↗ Weyl) →
"Evidence for events to have happened", given the future.

(4) Can QM predict frequencies (emp. probs.) of possible "histories", and how?

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5. Calculus of frequencies

(Lüders, Schwinger, Wigner)

$\{P_n, P_{n-1}, \dots, P_1\}$: time-ordered history of possible events,

$$P_i := P_{a^i_{t_i}}(I_i),$$

$a^i = (a^i)^* \in \mathcal{A}_S$, $t_0 < t_1 < t_2 < \dots < t_n$, $I_i \subset \mathbb{R}$,

given E, $\{\alpha_{t,s}\}$.

QM predicts "frequency"/empir. prob. of $\{P_n, P_{n-1}, \cdots, P_1\}$, given an initial "state" ω of SvE at t_0.

"Master formula."

$$\mathcal{F}_\omega\{P_n, \cdots, P_1\} := \omega\left(P_1 P_2 \cdots P_{n-1} P_n P_{n-1} \cdots P_2 P_1\right) \qquad (3)$$

Properties of $\mathcal{F}_\omega$:

(i) $\mathcal{F}_\omega\{P_n, \cdots, P_1\} \geq 0$

(ii) Set $P_j^1 := P_j$, $P_j^\alpha \cdot P_j^\beta = \delta^{\alpha\beta} P_j^\alpha$,

$$\sum_{\alpha=2}^{n_j} P_j^\alpha = \mathbb{1} - P_j, \quad n_j \geq 2.$$

$$\sum_{\underline{\alpha}} \mathcal{F}_\omega\{P_n^{\alpha_n}, \cdots, P_1^{\alpha_1}\} = 1$$

$$\Longrightarrow 0 \leq \mathcal{F}_\omega\{P_n, \cdots, P_1\} \leq 1 \qquad (4)$$

For "realistic" syst. S, $\mathcal{F}_\omega$ obeys a 0-1 Law if ω is pure.

(iii) "Symm. betw. prediction & retrodiction" – cycl. of tr (A-B-L)

(iv) (Non-) complementarity of "events"

$$\mathcal{F}_\omega\{P_n, \dots, P_j, \dots, P_1\} + \sum_{\alpha=2}^{n_j} \mathcal{F}_\omega\{P_n, \dots, P_j^\alpha, \dots, P_1\}$$

$$\underset{\text{i.g.}}{\neq} \mathcal{F}_\omega\{P_n, \dots, P_{j+1}, P_{j-1}, \dots, P_1\}, \quad (5)$$

unless $j = n$, because, i.g.,

$$\sum_{\alpha \neq \beta} \omega(P_1 \cdots P_j^\alpha \cdots P_n \cdots P_j^\beta \cdots P_1) \neq 0 \quad (6)$$

"interference"

$\Rightarrow$ For $n_j > 2$, no meaningful notion of "conditional probability" of P_j, given future; ($\rightarrow$ K-S!)

A possible event, P_j, can become a rec. fact iff it does not interfer w. compl. events, P_j^β, $\beta > 1$, given future events.

"Evidence" for P_j to happen:

$$\mathcal{E}_\omega(P_j / \{P_j\}^c) :=$$

$$1 - \max_{\{\alpha_k : k > j\}} \sum_{\alpha \neq \beta} \left| \omega(P_1 \cdots P_j^\alpha \cdots P_n^{\alpha_n} \cdots P_j^\beta \cdots P_1) \right|$$

History $\{P_n, \cdots, P_1\}$ is $(1-\delta)$-consistent w.r. to ω iff

$$\mathcal{E}_\omega(P_j / \{P_j\}^c) \geq 1 - \delta, \ \forall j. \qquad (7)$$

Only $\varkappa$-consistent histories, w. $0 \leq 1 - \varkappa \ll 1/n$, admit a meaningful "class." interpretation – FAPP.

Example ($n=2$): 2-slit exp. with screen & laser lamp.

How do $\varkappa$-consistent histories arise ($1-\varkappa \ll 1/n$)?

6. Dephasing & decoherence

Consider history $\{P_n, \dots, P_1\}$, w.

$$P_i := P_{a^i_{t_i}}(I_i),\ a^i = (a^i)^* \in \mathcal{A}_S,$$

$$I_i \subset \mathbb{R},\ t_n > t_{n-1} > \dots > t_1 > t_0.$$

At some time $t_< \geq t_{j-1}$, measurement of a^j starts (choice of E & of interactions $S \leftrightarrow E$ after time $t_<$), with possible outcome $P_j = P_{a^j_{t_j}}(I_j)$ at time $t_j > t_<$.

Under what conditions can P_j be considered a fact, given that, at times $t_n > \cdots > t_{j+1} > t_j$, events $P_n, \cdots, P_{j+1}$ may happen?

Events complementary to P_j:

$$P_j^\perp = \mathbb{1} - P_j = \sum_{\ell > 1} P_j^\ell$$

$\leftarrow$ spect. proj. of $a^j_{t_j}$.

ρ: State of $S \vee E$ at time $t_<$.

P_j a "fact" iff

$$F_\rho\{P_n, \cdots, P_{j+1}, P_j\} + \sum_{\ell \geq 1} F_\rho\{P_n, \cdots, P_{j+1}, P_j^\ell\}$$
$$\simeq F_\rho\{P_n, \cdots, P_{j+1}\} \qquad (8)$$

Sufficient condition for (8):

- $\overline{\mathcal{A}}_S := \langle b := \prod_i \alpha_{t_i, t_0}(a_i) \in \mathcal{B}_{S \vee E}, \; a_i \in \mathcal{A}_S, \forall i \rangle$

- $\tau_t : \overline{\mathcal{A}}_S \to \overline{\mathcal{A}}_S$ defined by

$$\tau_t(b) := \prod_i \alpha_{t_i + t, t_0}(a_i)$$

- Dephasing (given E, ρ)

$$\overline{\rho(\tau_\cdot(b))} = \overline{\rho(P_j \tau_\cdot(b) P_j)} + \sum_{\alpha \geq 2} \overline{\rho(P_j^\alpha \tau_\cdot(b) P_j^\alpha)},$$

in TD limit of E; where $\overline{(\cdot)}$ denotes erg. average over t.

For finite times & "finite" E, dephasing can be "undone" by ops. on E.

Sufficient for dephasing, but stronger, is:

<u>Decoherence</u> (given E):

$[\alpha_{t_j, t_0}(a_j), \tau_t(b)] \xrightarrow{w} 0,$

as $t \to \infty$ & in TD lim of E;

(e.g., τ_t "asy. abelian" on $\bar{\mathcal{A}}_S$).

Given only measurements at much later times, $t \to \infty$,

$\alpha_{t_j, t_0}(a_j)$ becomes <u>"central"</u>.

As $t \to \infty$, decoherence cannot be undone by ops. on E.

Assuming dephasing or decoherence, frequency of event P_j approaches $\rho(P_j)$, indep. of later measurements at much later times.

"Born's Rule"

"Almost classical ($\rightarrow$ consistent) histories" are close to "class. histories", in following sense:

Given hist. $\{P_n, \cdots, P_1\}$ with

$$\|[P_i, P_j]\| < \varepsilon, \ \forall i,j;$$

then $\exists\, C_n < \infty$ & hist. $\{\widetilde{P}_n, \cdots, \widetilde{P}_1\}$ s.t. $\widetilde{P}_i$ an orth. proj. with

$$\|\widetilde{P}_i - P_i\| < C_n \varepsilon, \ \forall i, \ \&$$

$$[\widetilde{P}_i, \widetilde{P}_j] = 0, \ \forall i,j.$$

Models for dephasing & decoherence: Primas, Coleman–<u>Hepp</u> (Fierz), Zeh, …, Dürr et al., Balian et al., J.F. et al., C.S., … :

Quantum theory of systems with ∞ many degs. of freedom.
